C3 DIFFERENTIATION — Answers - Worksheet J

1 $x = -1,\ y = 8$

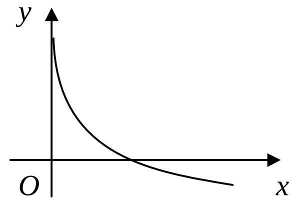
$\frac{dy}{dx} = \frac{3}{2}(3-x)^{\frac{1}{2}} \times (-1) = -\frac{3}{2}(3-x)^{\frac{1}{2}}$

grad $= -3$

$\therefore\ y - 8 = -3(x + 1)$

$[\ y = 5 - 3x\ ]$

2 **a**



b $y = 0\ \therefore\ x = \frac{1}{2}e^3$

$(\frac{1}{2}e^3, 0)$

c $x = 5\ \therefore\ y = 3 - \ln 10$

$\frac{dy}{dx} = -\frac{1}{x}$, grad $= -\frac{1}{5}$

$\therefore\ y - (3 - \ln 10) = -\frac{1}{5}(x - 5)$

$[\ y = -\frac{1}{5}x + 4 - \ln 10\ ]$

d at A, $y = 0\ \therefore\ x = 5(4 - \ln 10)$

at B, $x = 0\ \therefore\ y = 4 - \ln 10$

area $= \frac{1}{2} \times 5(4 - \ln 10) \times (4 - \ln 10)$

$= 7.20$ (3sf)

3 **a** $= 4(3x - 1)^3 \times 3$

$= 12(3x - 1)^3$

b $= \dfrac{2x \times \sin 2x - x^2 \times 2\cos 2x}{\sin^2 2x}$

$= \dfrac{2x(\sin 2x - x\cos 2x)}{\sin^2 2x}$

4 **a** $t = 3\ \therefore$ area $= 2e^{1.5} = 8.96\ \text{cm}^2$ (3sf)

b $\frac{dA}{dt} = 2 \times 0.5e^{0.5t} = e^{0.5t}$

$t = 3$, $\frac{dA}{dt} = e^{1.5} = 4.4817\ \text{cm}^2\ \text{yr}^{-1}$

$\therefore$ rate per day $= 4.4817 \div 365 = 0.0123$

area increasing at $0.0123\ \text{cm}^2$ per day (3sf)

c $65 = 2e^{0.5t}$

$t = 2 \ln 32.5 = 6.96$

$\therefore$ 7 years

d A increases exponentially and would become larger than the surface area of the boulder

5 **a** $\frac{dy}{dx} = \frac{a}{x} - 4$

SP: $\frac{a}{x} - 4 = 0$

$x = \frac{1}{4}a$

$\therefore\ (\frac{1}{4}a, a\ln\frac{a}{4} - a)$

b $x = 1\ \therefore\ y = -4$, grad $= a - 4$

$\therefore\ y + 4 = (a - 4)(x - 1)$

$[\ y = (a - 4)x - a\ ]$

c $(3, 0)\ \therefore\ 0 = 3(a - 4) - a$

$a = 6$

6 **a** $\frac{dy}{dx} = 2e^{2x}\sin x + e^{2x}\cos x$

$= e^{2x}(2\sin x + \cos x)$

b $\frac{d^2y}{dx^2} = 2e^{2x}(2\sin x + \cos x) + e^{2x}(2\cos x - \sin x)$

$= e^{2x}(3\sin x + 4\cos x)$

$\therefore\ \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y$

$= e^{2x}(3\sin x + 4\cos x)$

$\quad - 4e^{2x}(2\sin x + \cos x) + 5e^{2x}\sin x$

$= e^{2x}(3\sin x + 4\cos x - 8\sin x - 4\cos x + 5\sin x)$

$= 0$

7 **a** $\frac{dx}{dy} = 2\tan y \sec^2 y$

$= 2\tan y\,(\tan^2 y + 1)$

$= 2\sqrt{x}\,(x + 1)$

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{1}{2\sqrt{x}(x+1)}$

b $y = \frac{\pi}{4}$ $\therefore x = 1$

grad $= \frac{1}{4}$

$\therefore$ grad of normal $= -4$

$\therefore y - \frac{\pi}{4} = -4(x - 1)$

$[\ 16x + 4y - \pi - 16 = 0\ ]$

8 **a** $\frac{dy}{dx} = \frac{1\times\sqrt{x-2} - (x+2)\times\frac{1}{2}(x-2)^{-\frac{1}{2}}}{x-2}$

$= \frac{2(x-2)-(x+2)}{2(x-2)^{\frac{3}{2}}}$

$= \frac{x-6}{2(x-2)^{\frac{3}{2}}}$

b SP: $\frac{x-6}{2(x-2)^{\frac{3}{2}}} = 0$

$x = 6$ $\therefore (6, 4)$

c $x = 3$ $\therefore y = 5$, grad $= -\frac{3}{2}$

$\therefore$ grad of normal $= \frac{2}{3}$

$\therefore y - 5 = \frac{2}{3}(x - 3)$

$3y - 15 = 2x - 6$

$2x - 3y + 9 = 0$

9 **a** $= e^x \times (x - 1)^2 + e^x \times 2(x - 1) \times 1$

$= e^x(x^2 - 2x + 1 + 2x - 2)$

$= e^x(x^2 - 1)$

b $= e^x \times (x^2 - 1) + e^x \times 2x$

$= e^x(x^2 + 2x - 1)$

c SP: $e^x(x^2 - 1) = 0$

$x = \pm 1$

$\therefore (-1, 4e^{-1}), (1, 0)$

$(-1, 4e^{-1})$, $\frac{d^2y}{dx^2} = -2e^{-1}$ $\therefore$ maximum

$(1, 0)$, $\frac{d^2y}{dx^2} = 2e$ $\therefore$ minimum

d $x = 2$ $\therefore y = e^2$

grad $= 3e^2$

$\therefore y - e^2 = 3e^2(x - 2)$

$y = 3e^2x - 5e^2$

$y = e^2(3x - 5)$

10 **a** $\frac{dy}{dx} = x - \frac{3}{x}$

SP: $x - \frac{3}{x} = 0$

$x^2 = 3$

$x > 0$ $\therefore x = \sqrt{3}$

b $\frac{d^2y}{dx^2} = 1 + 3x^{-2}$

$x = \sqrt{3}$, $\frac{d^2y}{dx^2} = 2$

$\therefore$ minimum

c $y = \frac{1}{2}(\sqrt{3})^2 - 3\ln\sqrt{3}$

$= \frac{3}{2} - 3\ln 3^{\frac{1}{2}}$

$= \frac{3}{2} - \frac{3}{2}\ln 3$

$= \frac{3}{2}(1 - \ln 3)$

d $x = 1$ $\therefore y = \frac{1}{2}$

grad $= -2$

$\therefore y - \frac{1}{2} = -2(x - 1)$

$4x + 2y - 5 = 0$

11 **a** $f(x) = \frac{6x - 2(x+2)}{(x-1)(x+2)} = \frac{4x-4}{(x-1)(x+2)}$

$= \frac{4(x-1)}{(x-1)(x+2)} = \frac{4}{x+2}$

b $x = 2$ $\therefore y = 1$

$f'(x) = -4(x + 2)^{-2}$

grad $= -\frac{1}{4}$

$\therefore y - 1 = -\frac{1}{4}(x - 2)$

$4y - 4 = -x + 2$

$x + 4y = 6$